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# TABLE-TOP TIME-REVERSAL VIOLATION [1]

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## ABSTRACT

Many electrical and mechanical systems with two normal modes are appropriate for illustrating the quantum mechanics of neutral kaons. The illustration of CP- or time-reversal-violation in the neutral kaon system by mechanical or electrical analogues is more subtle. Some possibilities which could be realized in a table-top demonstration are suggested.

## I. INTRODUCTION

The problem of two coupled degenerate systems is one of the first a student encounters when learning about normal modes. In quantum mechanics, a fine example (see, e.g., Ref. [1]) is provided by the states of neutral kaons [2]. There exist two states $K^0$ and $\bar{K}^0$ which are degenerate as long as the product of charge conjugation $C$, space inversion $P$ (for parity), and time reversal $T$ leaves the theory invariant. (Lorentz-invariant local quantum field theories indeed are invariant under the combined product CPT [3].) In a theory invariant under CP [4], the linear combination of $K^0$ and $\bar{K}^0$ with $CP = +1$ couples to two pions, and is short-lived, while the other combination with $CP = -1$ does not, and is long-lived. There are many ways to illustrate this two-state system in classical physics, including vibrating membranes, coupled pendula, and coupled oscillators.

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The discovery of CP violation [5] indicated that the neutral kaon system is somewhat more complex than the typical two-state problem. Both the short-lived and long-lived neutral kaon can decay to $\pi\pi$. An interesting challenge is to illustrate this behavior in a table-top setting based on classical physics. We shall assume a CPT-invariant theory, so that it would suffice to illustrate either CP- or time-reversal-violation. In the present article we suggest some possibilities based on electrical circuits, utilizing effects in which the time-reversal violation is induced by means of an external magnetic field. Our purpose is to stimulate discussion of further illustrations, with a possible eye to understanding the way in which T-violation actually arises in Nature.

In Section II we describe three two-state systems: the neutral kaons (in a CP-invariant context), a coupled-pendulum analogue, and an electrical analogue. We then turn to the CP-violating problem for neutral kaons in Sec. III. Some electrical analogues of the CP-violating problem are suggested in Sec. IV, while Sec. V concludes.

## II. THE TWO-STATE SYSTEM IN A CP-CONSERVING THEORY

### A. Neutral kaons

In order to make sense of a class of "strange" particles produced strongly but decaying weakly, Gell-Mann and Nishijima [6] in 1953 proposed an additive quantum number, "strangeness," conserved in the strong interactions but not in the weak interactions. The reaction $\pi^- p \to K^0 \Lambda$, for example, would conserve strangeness $S$ if $S(\pi) = S(p) = 0$, $S(\Lambda) = -1$, and $S(K^0) = +1$. The kaon could not be its own antiparticle; there would have to also exist a $\bar{K}^0$ with $S(\bar{K}^0) = -1$. It could be produced, for example, in the reaction $\pi^- p \to K^0 \bar{K}^0 n$.

The states $K^0$ and $\bar{K}^0$ would be degenerate in the absence of coupling to final states (or to one another). However, both states can decay to the $2\pi$ final state in an S-wave (orbital angular momentum $\ell = 0$). Gell-Mann and Pais [2] noted that since $C(\pi^+\pi^-)_{\ell=0} = +$, $C(K^0) = \bar{K}^0$, and $C(\bar{K}^0) = K^0$, the linear combination of $K^0$ and $\bar{K}^0$ which decayed to $\pi^+\pi^-$ had to be $K_1 \equiv (K^0 + \bar{K}^0)/\sqrt{2}$. Then there should be another state $K_2 \equiv (K^0 - \bar{K}^0)/\sqrt{2}$ forbidden to decay to $\pi^+\pi^-$, and thus long-lived. (It should be able to decay to $3\pi$, for example.) This state was looked for and found [7]. Its lifetime was measured to be about 600 times that of $K_1$.

Gell-Mann and Pais assumed that C was conserved in the weak decay process. In a CP-conserving weak interaction theory in which C and P are individually violated, the above argument can be recovered by replacing C with CP [8]. If one chooses the phase of the particle states in such a way that $\bar{K}^0 = CPK^0$, the eigenstates with positive and negative CP are then, as before,

$$K_1 = \frac{K^0 + \bar{K}^0}{\sqrt{2}} \quad , \quad K_2 = \frac{K^0 - \bar{K}^0}{\sqrt{2}} \quad . \tag{1}$$

### B. Mechanical analogues

The $K^0 - \bar{K}^0$ system resembles many coupled degenerate problems in classical physics. For example, a drum-head in its first excited state possesses a line of nodes. A degenerate state exists with the line of nodes perpendicular to the first, but nothing specifies the absolute orientations of the two lines of nodes. However, if a fly alights off-center on the drum, it will define the two lines of nodes. One mode will couple to the fly, thereby changing in frequency, and one mode will not.

A system of two coupled pendula [9] also provides a simple analogy to the $K_1 - K_2$ system. The requirement of CPT invariance is satisfied by taking the two pendula to have equal natural frequencies $\omega_0$ (and, for simplicity, equal lengths and masses). If one couples them by a connecting spring, the two normal modes will consist of one with frequency $\omega_1 = \omega_0$ in which the two pendula oscillate in phase, and another with $\omega_2 > \omega_0$ in which the two oscillate 180° out of phase, thereby compressing and stretching the connecting spring. If the connection dissipates energy, the mode with frequency $\omega_2$ will eventually decay away, leaving only the in-phase mode with frequency $\omega_1 = \omega_0$.

### C. Electrical analogue

A simple electrical analogue of the neutral kaon system can be constructed using two $L - C$ "tank" circuits, each consisting of a capacitor $C_i$ in parallel with an inductor $L_i$ and having resonant frequency $\omega_{0i} = (L_i C_i)^{-1/2}$ $(i = 1, 2)$. Currents $I_1$ and $I_2 = -I_1$ flow through each circuit to ground. The two tank circuits are coupled through an impedance $Z_c$, so that the voltages on each circuit are related to the currents by $V_2 - V_1 = Z_c I_1 = Z_c(I_1 - I_2)/2$. The network is shown in Fig. 1.

Adding the currents flowing through each inductor and capacitor, $I_i = I_{Li} + I_{Ci}$, and noting that $L_i \dot{I}_{Li} = V_i$ and $I_{Ci} = C_i \dot{V}_i$, we find $\dot{I}_i = V_i/L_i + C_i \ddot{V}_i$. Thus

$$\dot{V}_2 - \dot{V}_1 = \frac{Z_c}{2}\left[\frac{V_1}{L_1} + C_1\ddot{V}_1 - \frac{V_2}{L_2} - C_2\ddot{V}_2\right] \quad . \tag{2}$$

Moreover, since $I_1 + I_2 = 0$, we can write

$$\frac{V_1}{L_1} + C_1\ddot{V}_1 + \frac{V_2}{L_2} + C_2\ddot{V}_2 = 0 \quad . \tag{3}$$

We have written the equations in a form which exhibits the symmetry between the tank circuits 1 and 2. Now we assume harmonic behavior: $V_i = v_i e^{-i\omega t}$, and solve the following coupled equations in $v_1$ and $v_2$ for characteristic values of $\omega$:

$$\left(\frac{1}{L_1} - \omega^2 C_1 - 2iY_c\omega\right) v_1 - \left(\frac{1}{L_2} - \omega^2 C_2 - 2iY_c\omega\right) v_2 = 0 \quad , \tag{4}$$

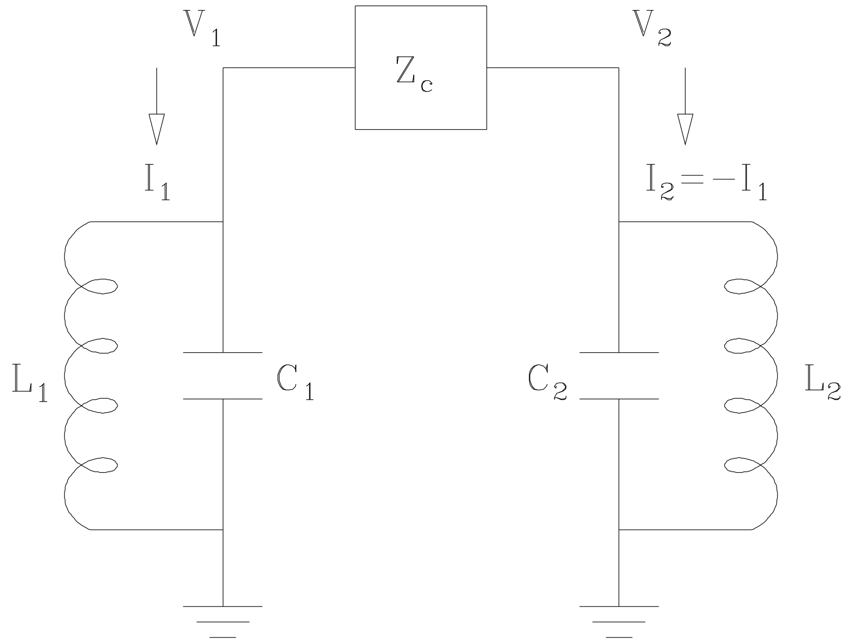


Figure 1: Coupled "tank" circuits illustrating the $K^0 - \bar{K}^0$ system.

$$\left(\frac{1}{L_1} - \omega^2 C_1\right) v_1 + \left(\frac{1}{L_2} - \omega^2 C_2\right) v_2 = 0 \quad . \tag{5}$$

Here we have defined the coupling admittance as the reciprocal of the coupling impedance: $Y_c \equiv Z_c^{-1}$.

When $Y_c = 0$, if the coefficient of $v_i$ is zero but that of $v_j$ $(j \neq i)$ is nonzero, then we have a solution with $v_i \neq 0$ and $v_j = 0$. Now, however, let us assume the natural frequencies of the two tank circuits are the same: $L_1 C_1 = L_2 C_2 = \omega_0^{-2}$. After some simplification, the characteristic equation reduces to

$$(\omega_0^2 - \omega^2)\left[1 - \frac{\omega^2}{\omega_0^2} - iY_c\omega(L_1 + L_2)\right] = 0 \quad . \tag{6}$$

One solution has the natural frequency $\omega = \omega_0$ independently of the coupling. This is the solution with $v_1 = v_2$, where no current flows through the coupling device. The other solution has a frequency shift which in general has both real and imaginary parts. Defining $\delta\omega \equiv \omega - \omega_0$, we find $\delta\omega/\omega \approx -2iY_c(L_1 + L_2)\omega$. A real admittance (corresponding to a resistive coupling) leads to an imaginary frequency shift, and a damping of the oscillation. An imaginary admittance (corresponding to an inductive or capacitive coupling) leads to a real frequency shift.

The solution with $v_1 = v_2$ corresponds in the neutral kaon system to the $K_2$ state, which does not couple to two pions. The solution with $v_1 \neq v_2$ corresponds to the $K_1$ state, which decays to two pions and is shifted in both mass and width from the $K_2$.

## III. NEUTRAL KAONS WITH CP VIOLATION

The discovery [5] in 1964 that both the short-lived "$K_1$" and long-lived "$K_2$" states decayed to $\pi\pi$ upset the tidy picture of a two-state system described in Sec. II A. It signified that not even CP symmetry was valid in Nature. Henceforth the states of definite mass and lifetime would be known as $K_S$ (for "short") and $K_L$ (for "long"). They can be parametrized approximately as

$$|S\rangle \simeq |K_1\rangle + \epsilon|K_2\rangle \quad , \quad |L\rangle \simeq |K_2\rangle + \epsilon|K_1\rangle \quad , \tag{7}$$

where we shall use the shorthand $S,\ L$ for $K_S,\ K_L$. The complex parameter $\epsilon$ encodes all we know at present about CP violation in the neutral kaon system. Its magnitude is $\epsilon = (2.26 \pm 0.02) \times 10^{-3}$ and its phase is approximately $45^\circ$. Notice that, in contrast to $|K_1\rangle$ and $|K_2\rangle$, the states $|S\rangle$ and $|L\rangle$ are not orthogonal to one another but have a scalar product $\langle L|\rangle S \approx 2\ \mathrm{Re}\ \epsilon$.

A convenient way to discuss the above problem is to express $K_S$ and $K_L$ as eigenstates of a $2 \times 2$ "mass matrix" $\mathcal{M}$ [10, 11]. In the kaon rest frame, the time evolution of basis states $K^0$ and $\bar{K}^0$ can be written[12] as

$$i\frac{\partial}{\partial t}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = \mathcal{M}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} \quad ; \quad \mathcal{M} = M - i\Gamma/2 \quad . \tag{8}$$

An arbitrary matrix $\mathcal{M}$ can be written in terms of Hermitian matrices $M$ and $\Gamma$. CPT invariance can be shown to imply the restriction $\mathcal{M}_{11} = \mathcal{M}_{22}$ and hence $M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22}$. We adopt this limitation here.

We denote the eigenstates of $\mathcal{M}$ by

$$|S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \tag{9}$$

$$|L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \tag{10}$$

with $|p|^2 + |q|^2 = 1$, and the corresponding eigenvalues by $\mu_{S,L} \equiv m_{S,L} - \frac{i}{2}\Gamma_{S,L}$, where $m_{S,L}$ and $\Gamma_{S,L}$ are real. Here we have taken the condition $\mathcal{M}_{11} = \mathcal{M}_{22}$ into account. With $\epsilon \equiv (p-q)/(p+q)$, we can write

$$|S\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right] \quad , \tag{11}$$

$$|L\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right] \quad . \tag{12}$$

Expressing $K^0$ and $\bar{K}^0$ in terms of the CP eigenstates by means of Eq. (1), we recover the relation (7).

One can relate $\epsilon$ to the properties of the mass matrix and mass eigenvalues. Making a phase choice, we can write

$$\frac{q}{p} = \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} \tag{13}$$

and note that

$$\mu_S = \mathcal{M}_{11} + \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad ; \quad \mu_L = \mathcal{M}_{11} - \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad , \tag{14}$$

so

$$\mu_S - \mu_L = 2\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad . \tag{15}$$

Then

$$\epsilon = \frac{p-q}{p+q} = \frac{\sqrt{\mathcal{M}_{12}} - \sqrt{\mathcal{M}_{21}}}{\sqrt{\mathcal{M}_{12}} + \sqrt{\mathcal{M}_{21}}} \simeq \frac{\mathcal{M}_{12} - \mathcal{M}_{21}}{4\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}} \quad , \tag{16}$$

where the smallness of $\epsilon$ has been used. With the definition of $\mathcal{M}$ and (15) we can then write

$$\epsilon \simeq \frac{\mathrm{Im}(\Gamma_{12}/2) + i\ \mathrm{Im}M_{12}}{\mu_S - \mu_L}, \tag{17}$$

so that the CP-violation parameter $\epsilon$ arises from imaginary parts of off-diagonal terms in the mass matrix.

The matrices $\Gamma$ and $M$ may be expressed [13] in terms of sums over states connected to $K^0$ and $\bar{K}^0$ by the weak Hamiltonian $H_W$. By considering specific $2\pi$, $3\pi$, $\pi l \nu$, and other final states, one can show that $|\mathrm{Im}\Gamma_{12}/2| \ll |\mathrm{Im}M_{12}|$. This result then implies a specific phase of $\epsilon$:

$$\mathrm{Arg}\ \epsilon \approx \left\{ \begin{array}{c} 90^0 \\ 270^0 \end{array} \right\} - \ \mathrm{Arg}(\mu_S - \mu_L) \ \ \text{for} \ \ \left\{ \begin{array}{c} \mathrm{Im}M_{12} > 0 \\ \mathrm{Im}M_{12} < 0 \end{array} \right\} \tag{18}$$

Given the measurements [14] $m_S - m_L = -0.476\ \Gamma_S$, $\Gamma_S - \Gamma_L = 0.998\ \Gamma_S$, we have $\mu_S - \mu_L = -(0.476 + 0.499i)\Gamma_S$, or Arg $(\mu_S - \mu_L) = (3\pi/2) -$ arctan $(0.476/0.499) = (3\pi/2) - 43.6^0$. Thus

$$\mathrm{Arg}\ \epsilon = (43.6 \pm 0.2)^0 \ \ (\mathrm{Im}\ M_{12} < 0) \quad ,$$

$$\mathrm{Arg}\ \epsilon = \pi + (43.6 \pm 0.2)^0 \ \ (\mathrm{Im}\ M_{12} > 0) \quad . \tag{19}$$

We seek in classical physics an analogue of the two-state mixing problem which leads to a non-zero value of $\epsilon$. Equation (16) implies that $\epsilon \neq 0$ arises from a lack of symmetry of the mass matrix $\mathcal{M}$. There may be additional manifestations of CP violation in kaon decays not reflected in the parameter $\epsilon$ (for example, if the amplitude ratios $A(L \to \pi\pi)/A(S \to \pi\pi)$ differ for charged and neutral pions), but experiments [15, 16] do not yet conclusively demonstrate their presence.

## IV. ELECTRICAL ANALOGUES OF T-VIOLATION

### A. Mass matrix formulation

In order to recast the electrical problem in a form closer to that of Sec. III, we rewrite Eqs. (4) and (5) as

$$\mathcal{M}\mathbf{v} = \omega^2 \mathbf{v} \quad , \tag{20}$$

where

$$\mathcal{M} \equiv \begin{bmatrix} \omega_0^2 - \frac{iY_c\omega_0}{C_1} & \frac{iY_c\omega_0}{C_1} \\ \frac{iY_c\omega_0}{C_2} & \omega_0^2 - \frac{iY_c\omega_0}{C_2} \end{bmatrix} \quad , \tag{21}$$

and

$$\mathbf{v} \equiv \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad . \tag{22}$$

Here we have used the earlier assumption $L_1C_1 = L_2C_2 = \omega_0^{-2}$, and have replaced $\omega$ by $\omega_0$ in the coupling term.

Note that "CPT invariance," with $\mathcal{M}_{11} = \mathcal{M}_{22}$, is not necessarily a feature of this mass matrix. We shall assume it to be so by taking $C_1 = C_2 \equiv C$ for simplicity.

The violation of "CP" or "T" invariance is now parametrized by allowing the presence of small antisymmetric off-diagonal terms in $\mathcal{M}$:

$$\mathcal{M}_{12} \to \mathcal{M}_{12} - a\omega_0^2 \quad , \quad \mathcal{M}_{21} \to \mathcal{M}_{21} + a\omega_0^2 \quad . \tag{23}$$

We thereby parametrize an off-diagonal coupling which is not symmetric between $1 \to 2$ and $2 \to 1$.

The characteristic equation now becomes

$$(\omega_0^2 - \omega^2)^2 - 2i\frac{Y_c\omega_0}{C}(\omega_0^2 - \omega^2) + a^2\omega_0^4 = 0 \quad . \tag{24}$$

The solutions are modes with $\omega^2 \simeq \omega_0^2 - 2iY_c\omega_0/C$, corresponding to $K_S$ (whose mass is affected strongly by the "CP-conserving" coupling $Y_c$), and with $\omega^2 \simeq \omega_0^2 + (i/2)(aC\omega_0/Y_c)^2$, corresponding to $K_L$ (whose mass now also receives a small contribution from the "CP-violating" coupling $a$). The eigenstates can be written in the form (11) and (12), if we make the identification

$$\epsilon \simeq \frac{iaC\omega_0}{2Y_c} \quad . \tag{25}$$

Here we have assumed $a$ is sufficiently small so that $|\epsilon| \ll 1$. The relative strength and phase of the antisymmetric and symmetric couplings thus governs $\epsilon$, just as in the case of neutral kaons.

It is not obvious that a physical system can be constructed along the above lines without violating "CPT" invariance, since one might expect different terms

to appear in $\mathcal{M}_{11}$ and $\mathcal{M}_{22}$. The construction of an explicit "CPT-invariant" but "CP-violating" system so far has eluded us, but we now give some possibilities.

### B. Propagation of VLF radio waves in the ionosphere

It has been found [17] that radio waves of very low frequencies (in the 10 – 20 kHz range) propagate with different phase velocities and attenuations from west to east and east to west in the daytime ionosphere. This behavior has been traced to interaction with electron orbits in the Earth's magnetic field. The variations in attenuation can amount to differences of nearly 50%. This suggests that one might devise a table-top version of non-reciprocal behavior in which the coupling unit denoted by the box in Fig. 1 transfers energy differently from tank circuit 1 to 2 and from circuit 2 to 1. We now suggest two ways of implementing this idea.

### C. Faraday rotation

Let us imagine the tank circuits 1 and 2 in Fig. 1 to consist of two oscillators with identical freqencies, coupling to each other by means of half-wave dipole antennas whose planes of polarization are perpendicular to the line between them. Let each dipole be in the far (radiation) field of the other for simplicity. As long as the dipoles are parallel to one another, the energy transfer between the circuits is maximal. If the dipoles make an angle $\phi$ with respect to one another, the power transfer will be proportional to $\cos^2\phi$.

Now let the dipoles be separated by a medium which causes the radio waves to undergo Faraday rotation. This may be achieved by letting a magnetic field be present parallel to the line joining the two dipoles, and propagating the radio-frequency energy through a plasma such as the earth's ionosphere. Suppose that the medium causes a rotation of the plane of polarization by an angle $\psi$ to the right. This rotation will then be characteristic of radiation traveling both from circuit 1 to circuit 2 and from 2 to 1. If antenna 2 is oriented with respect to 1 by an angle $\phi$, then the power transfer from 1 to 2 will be proportional to $\cos^2(\phi - \psi)$, while that from 2 to 1 will be proprtional to $\cos^2(\phi + \psi)$. If, for example, $\phi = \psi = \pi/4$, power will be transfered with maximum efficiency from 1 to 2, but no power will flow from 2 to 1.

The presence of an external magnetic field with an orientation from 1 to 2 explicitly violates time-reversal invariance, so it is no surprise that one can induce an asymmetric mixing between the two circuits. The realization of this system in a practical demonstration would be of some interest. In analogy to the mass eigenstates $|S\rangle$ and $|L\rangle$ of the neutral kaon system, the eigenmodes would not be orthogonal to one another, and neither eigenmode would correspond to a solution with $v_1 = v_2$.

### D. Other possibilities

Any coupling term which simulates the behavior $\mathcal{M}_{12} \neq \mathcal{M}_{21}$ in the kaon mass matrix would be suitable for inducing an effect equivalent to $\epsilon \neq 0$. One could imagine utilizing, for example, the Hall effect, in which conduction from point 1 to point 2 would not necessarily be the same as that from point 2 to point 1 in the presence of a suitable magnetic field. Another possibility would be to simply insert an amplifier between the two tank circuits in Fig. 1. In this case it is not as clear how time-reversal invariance is violated.

## V. CONCLUSIONS

The demonstration of CP or time-reversal violation in a classical table-top setting would be an interesting and instructive extension of the problem of two coupled degenerate systems, such as pendula or oscillators. A coupling term which is asymmetric with respect to the two systems is required. We have suggested some ways in which this coupling might be realized in practice. The experience encountered in realizing such a circuit in a practical device might well offer insights into the way in which CP- or time-reversal invariance violation is actually realized in the neutral kaon system.

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# References

[1] G. Baym, *Lectures on Quantum Mechanics* (Benjamin, New York, 1969).

[2] M. Gell-Mann and A. Pais, "Behavior of neutral particles under charge conjugation," Phys. Rev. **97**, 1387-9 (1955).

[3] J. Schwinger, "The theory of quantized fields. II," Phys. Rev. **91**, 713-28 (1953) (see esp. p. 720 ff); "The theory of quantized fields. VI," Phys. Rev. **94**, 1362-84 (1954) (see esp. Eq (54) on p. 1366 and p. 1376 ff); G. Lüders, "On the equivalence of invariance under time reversal and under particle-antiparticle conjugation for relativistic field theories," Kong. Danske Vid. Selsk., Matt-fys. Medd. **28**, No. 5, 1-17 (1954); "Proof of the TCP theorem," Ann. Phys.

(N.Y.) **2**, 1-15 (1957); W. Pauli, "Exclusion principle, Lorentz group, and reflection of space-time and charge," in W. Pauli, ed. *Niels Bohr and the Development of Physics* (Pergamon, New York, 1955), pp. 30-51.

[4] R. P. Feynman and M. Gell-Mann, "Theory of the Fermi interaction," Phys. Rev. **109**, 193-8 (1958); E. C. G. Sudarshan and R. E. Marshak, "Chirality invariance and the universal Fermi interaction," Phys. Rev. **109**, 1860-2 (1958).

[5] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, "Evidence for the $2\pi$ decay of the $K_2^0$ meson," Phys. Rev. Lett. **13**, 138-40 (1964).

[6] M. Gell-Mann, "Isotopic spin and new unstable particles," Phys. Rev. **92**, 833-4 (1953); "On the Classification of Particles" (1953, unpublished); M. Gell-Mann, "The interpretation of the new particles as displaced charge multiplets," Nuovo Cim. **4**, Suppl. 848-66 (1956); T. Nakano and K. Nishijima, "Charge independence for $V$-particles," Prog. Theor. Phys. **10**, 581-2 (1953); K. Nishijima, "Some remarks on the even-odd rule," Prog. Theor. Phys. **12**, 107-8 (1954); "Charge independence theory of $V$ particles," Prog. Theor. Phys. **13**, 285-304 (1955).

[7] K. Lande, E. T. Booth, J. Impeduglia, and L. M. Lederman, "Observation of long-lived $V$ particles," Phys. Rev. **103**, 1901-4 (1956).

[8] T. D. Lee, R. Oehme and C. N. Yang, "Remarks on possible noninvariance under time reversal and charge conjugation," Phys. Rev. **106**, 340-5 (1957); B. L. Ioffe, L. B. Okun' and A. P. Rudik, "The problem of parity non-conservation in weak interactions," Zh. Eksp. Teor. Fiz. **32**, 396-7 (1957) [Sov. Phys. - JETP **5**, 328-30 (1957)].

[9] B. Winstein, "CP violation," in *Festi-Val – Festschrift for Val Telegdi*, ed. by K. Winter (Elsevier, Amsterdam, 1988), pp. 245-265.

[10] T. P. Cheng and L. F. Li, *Gauge Theory of Elementary Particles* (Oxford University Press, 1984).

[11] See, e.g., K. Kleinknecht, "CP violation in the $K^0 - \bar{K}^0$ system," in *CP Violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989), pp. 41-104, and references therein.

[12] See, e.g., R. G. Sachs, *The Physics of Time Reversal Invariance* (University of Chicago Press, Chicago, 1988).

[13] See, e.g., P. K. Kabir, *The CP Puzzle* (Academic Press, New York, 1968).

[14] Particle Data Group, L. Montanet *et al.*, "Review of particle properties," Phys. Rev. D **50**, 1173-1826 (1994).

[15] Fermilab E731 Collaboration, L. K. Gibbons *et al.*, "Measurement of the CP violation parameter $\mathrm{Re}(\epsilon'/\epsilon)$," Phys. Rev. Lett. **70**, 1203-6 (1993).

[16] CERN NA31 Collaboration, G. D. Barr *et al.*, "A new measurement of direct CP violation in the neutral kaon system," Phys. Lett. B **317**, 233-42 (1993).

[17] K. Davies, *Ionospheric Radio* (Peter Peregrinus Ltd., London, 1990), p. 378.